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Seiberg-Witten map for superfields on $N = (1/2, 0)$ and $N = (1/2, 1/2)$ deformed superspace

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Abstract: In this paper we construct the Seiberg-Witten maps for superfields on the $\theta - \theta$ deformed superspaces with $N = (1/2, 0)$ and $N = (1/2, 1/2)$ supersymmety. We show that on the $N = (1/2, 0)$ deformed superspace there is no Seiberg-Witten map for antichiral superfields which is at the same time antichiral, local and which preserves the $N = (1/2, 0)$ supersymmetry. Solutions which break this requirements are presented. On the $N = (1/2, 1/2)$ deformed superspace we show that for the chiral gauge parameter, and therefore also for the chiral matter field, there is no chiral Seiberg-Witten map. Some other possible Seiberg-Witten maps for the superfields are presented.

Keywords: Gauge Symmetry, Superspaces, Non-Commutative Geometry.

Contents

1. Introduction

Field theories on canonically deformed space

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}\,, \qquad \theta^{ij} = -\vartheta^{ji} \in \mathbb{R}\,, \tag{1.1}$$

have recently attracted much attention (for reviews and an exhaustive list of references see [1, 2, 3]), mainly due to the discovery of this noncommutative space in string theory [4, 5, 6]. Based on the existence of different regularization procedures in string theory, Seiberg and Witten claimed in [6] that certain noncommutative gauge theories are equivalent to commutative ones. In particular, they argued that there exists a map from a commutative gauge field to a noncommutative one, which is compatible with the gauge structure of each. This map has become known as the Seiberg-Witten map.

In [7]–[10] gauge theory on noncommutative space was formulated using the Seiberg-Witten map. In contrast to earlier approaches [12]–[15], this method works for arbitrary gauge groups. Using this method the problems of charge quantization [16, 17] and tensor product of gauge groups [15] were solved and the standard model and GUT's were formulated at the tree level on noncommutative space [18, 19].

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Non(anti)commutative superspaces naturally arise in string theory as well with $x-x$ deformation (canonical deformation) [20], $\theta-\theta$ deformation [21, 22, 23] and $x-\theta$ deformation [24]. General deformed superspaces were first studied more closely in [25, 26] and recently in [27] and in connection with the supermatrix model in [28, 29].

Since the work of Seiberg [23], various aspects of field theory were considered on the $\theta-\theta$ deformed superspace with $N=(1/2,0)$ supersymmetry. Renormalization properties of the Wess-Zumino model and gauge theories were considered in [30]–[37]. Solitons, instanton solutions and some nonpertubative aspects were considered in [38]–[43]. The generalization to $N=2$ and other interesting features have been explored in [44]–[56].

Till now, on the $\theta-\theta$ deformed superspace gauge theories were defined without Seiberg-Witten map. Similar to the nonsupersymmetric case, without the Seiberg-Witten map only gauge theories for the U(N) gauge group can be formulated [45, 42]. Hence, in order to consider arbitrary gauge groups we are forced to determine the Seiberg-Witten maps for superfields.

In [57] it was shown, that on canonically deformed $N=1$, $d=4$ euclidean superspace there exist a local, chiral and supersymmetric Seiberg-Witten map for chiral superfields if we take the noncommutativity parameter to be selfdual. If the noncommutativity parameter is antiselfdual, there exist a local, antichiral and supersymmetric Seibeerg-Witten map for antichiral superfields. Furthermore, it was shown that on the canonically deformed $N=1$, $d=4$ Minkowski superspace the Seiberg-Witten map is not compatible with locality, (anti)chirality and supersymmetry at the same time.

It is an interesting question, if we face the same problems on the $\theta-\theta$ deformed superspace. The aim of this paper is to answer this question. First we will recapitulate some well known properties of the $N=(1/2,0)$ deformed euclidean superspace [23] and construct the Seiberg-Witten maps in terms of component fields just to get some feeling for the problems which will occur. Thereafter we will construct the Seiberg-Witten maps in terms of superfields and discuss briefly their properties. Finally we make the same considerations on the $N=(1/2,1/2)$ deformed superspace [27, 54]. We use the conventions of [58].

2. $N=(1/2,0)$ deformed euclidean superspace

We consider the following deformed superspace [23]:

$$\begin{aligned} & [\hat{x}^i,\hat{x}^j] = \bar{\theta}\bar{\theta}C^{ij}\,, \\ \hat{\mathcal{R}}:\; & [\hat{x}^i,\hat{\theta}^\alpha] = iC^{\alpha\beta}\sigma^m_{\beta\dot\beta}\bar{\hat{\theta}}^{\dot\beta}\,, \qquad [\hat{x}^i,\bar{\hat{\theta}}^{\dot\beta}]=0\,, \\ & \{\hat{\theta}^\alpha,\hat{\theta}^\beta\} = C^{\alpha\beta}\,, \qquad \{\bar{\hat{\theta}}^{\dot\alpha},\bar{\hat{\theta}}^{\dot\beta}\} = \{\hat{\theta}^\alpha,\bar{\hat{\theta}}^{\dot\alpha}\} = 0\,, \end{aligned} \tag{2.1}$$

where $C^{\alpha\beta}$ is a constant, symmetric deformation parameter and

$$C^{ij} = C^{\alpha\beta}\varepsilon_{\beta\gamma}(\sigma^{ij})_\alpha{}^\gamma \tag{2.2}$$

is selfdual. Useful identities are

$$C^{\alpha\beta} = \frac{1}{2}\,\varepsilon^{\alpha\gamma}(\sigma^{ij})_\gamma{}^\beta C_{ij}\,, \tag{2.3}$$

$$|C|^2 = C^{ij}C_{ij} = 4\det C\,. \tag{2.4}$$

Non(anti)commutativity is indicated by a hat. On this deformed superspace $\hat{\bar{\theta}}$ is not the complex conjugate of $\hat{\theta}$, which is possible only in euclidean space. We will be working on euclidean $\hat{\mathbb{R}}^4$, but we will continue to use the lorentzian signature notation.

Using $y^m = x^m + i\theta\sigma^m\bar{\theta}$ we can accompany (2.1) with

$$\begin{aligned} & [\hat{y}^i, \hat{y}^j] = 0\,, \\ \hat{\mathcal{R}}:\ & [\hat{y}^i, \hat{\theta}^\alpha] = 0\,, \qquad [\hat{x}^i, \hat{\bar{\theta}}^{\dot\beta}] = 0\,, \\ & \{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}\,, \qquad \{\hat{\bar{\theta}}^{\dot\alpha}, \hat{\bar{\theta}}^{\dot\beta}\} = \{\hat{\theta}^\alpha, \hat{\bar{\theta}}^{\dot\alpha}\} = 0\,. \end{aligned} \tag{2.5}$$

Thus it is obvious to consider the deformed chiral superspace (2.5) instead of (2.1). Noncommutative functions and fields are defined as elements of the noncommutative algebra

$$\hat{\mathcal{A}} = \frac{\mathbb{C}[[\hat{y}^i, \hat{\theta}_\alpha, \hat{\bar{\theta}}_{\dot\alpha}]]}{I_{\hat{\mathcal{R}}}}\,, \tag{2.6}$$

where $I_{\hat{\mathcal{R}}}$ is the two-sided ideal created by the relations (2.5).

The derivatives act on the coordinates as in the classical case

$$[\hat{\partial}_i, \hat{y}^j] = \delta_i^{\ j}\,, \qquad \{\hat{\partial}_\alpha, \hat{\theta}^\beta\} = \delta_\alpha^{\ \beta}\,, \qquad \ldots \tag{2.7}$$

If nothing else is said ∂_i is the derivative according to the coordinate y^i. When $C^{\alpha\beta}$ is invertible, the fermionic derivatives ∂_α are internal operations in the algebra $\hat{\mathcal{A}}$ and they are given by[1]

$$\hat{\partial}_\alpha \hat{F}(\hat{x}, \hat{\theta}, \hat{\bar{\theta}}) = C^{-1}_{\alpha\beta}\left(\hat{\theta}^\beta \hat{F} - (-1)^{p(\hat{F})}\hat{F}\hat{\theta}^\beta\right), \tag{2.8}$$

which leads to the classical relation

$$\{\hat{\partial}_\alpha, \hat{\partial}_\beta\} = 0\,. \tag{2.9}$$

2.1 Symmetries

The algebra (2.1) is covariant under the group of classical supertranslations parameterized by $(\hat{a}, \hat{\xi}, \hat{\bar{\xi}})$

$$\begin{aligned} \hat{x}'^m &= \hat{x}^m + \hat{a}^m + i\hat{\theta}\sigma^m\hat{\bar{\xi}} - i\hat{\xi}\sigma^m\hat{\bar{\theta}}\,, \\ \hat{\theta}'_\alpha &= \hat{\theta}_\alpha + \hat{\xi}_\alpha\,, \\ \hat{\bar{\theta}}'_{\dot\alpha} &= \hat{\bar{\theta}}_{\dot\alpha} + \hat{\bar{\xi}}_{\dot\alpha}\,. \end{aligned} \tag{2.10}$$

which is generated by the complex charges $\hat{Q}_\alpha$, $\hat{\bar{Q}}_{\dot\alpha}$ and the four momentum $\hat{P}_m$. On the deformed chiral superspace (2.5) the generators have the form

$$\hat{P}_m = i\hat{\partial}_m\,, \tag{2.11}$$

$$\hat{Q}_\alpha = \hat{\partial}_\alpha\,, \tag{2.12}$$

$$\hat{\bar{Q}}_{\dot\alpha} = -\hat{\bar{\partial}}_{\dot\alpha} + 2i(\hat{\theta}\sigma^m)_{\dot\alpha}\hat{\partial}_m\,, \tag{2.13}$$

[1] $p(\hat{F}) = \begin{cases} 0: \hat{F} \text{ bosonic} \\ 1: \hat{F} \text{ fermionic} \end{cases}$

and satisfy following deformed supersymmetry algebra

$$
\begin{aligned}
[\hat{P}_m, \hat{P}_n] &= [\hat{P}_m, \hat{Q}_\alpha] = [\hat{P}_m, \hat{\bar{Q}}_{\dot\beta}] = 0\,, && (2.14)\\
\{\hat{Q}_\alpha, \hat{Q}_\beta\} &= 0\,, && (2.15)\\
\{\hat{Q}_\alpha, \hat{\bar{Q}}_{\dot\beta}\} &= 2\,\sigma^m_{\alpha\dot\beta}\,\hat{P}_m\,, && (2.16)\\
\{\hat{\bar{Q}}_{\dot\alpha}, \hat{\bar{Q}}_{\dot\beta}\} &= 4C^{\alpha\beta}\sigma^m_{\alpha\dot\alpha}\sigma^n_{\beta\dot\beta}\hat{P}_m\hat{P}_n\,. && (2.17)
\end{aligned}
$$

2.2 Star product

It is convenient to use the star product formulation of the algebra. This means that we use the commutative coordinates and functions but replace the ordinary product by the star product. The star product on this deformed superspace is

$$
\begin{aligned}
F(\theta) * G(\theta) &= \exp\left(-\frac{1}{2}C^{\alpha\beta}\partial^F_\alpha\partial^G_\beta\right) F(\theta)\,G(\theta)\\
&= FG - \frac{1}{2}(-1)^{p(F)}C^{\alpha\beta}\partial_\alpha F\,\partial_\beta G - \det C^{\alpha\beta}\frac{\partial}{\partial(\theta\theta)}F\frac{\partial}{\partial(\theta\theta)}G\,. \qquad (2.18)
\end{aligned}
$$

∂^F_α act only on F and ∂^G_β act only on G, e.g.

$$\partial^G_\beta(FG) = (-1)^{p(F)}\partial_\beta G\,. \qquad (2.19)$$

Also, similar to the case of the Weyl-Moyal star product [59, 60] one can show that

$$\int d^2\theta F(\theta) * G(\theta) = \int d^2\theta F(\theta)G(\theta)\,, \qquad (2.20)$$

which correspond to the fact that the difference

$$F(\theta) * G(\theta) - F(\theta)G(\theta) = \partial_\alpha(\cdots)^\alpha \qquad (2.21)$$

is a total Grassmann derivative not surviving the Grassmann integration.

The star product (2.18) is invariant under Q and therefore we expect it to be a symmetry of the space. However, since $\bar{Q}$ depends explicitely on θ, it is clear that the star product is not invariant under $\bar{Q}$. Therefore, $\bar{Q}$ is not a symmetry of the noncommutative space. Since half of the $N = (1/2, 1/2)$ supersymmetry is broken, we can refer to the unbroken Q supersymmetry as $N = (1/2, 0)$ supersymmetry.

2.3 Covariant derivatives and (anti)chiral superfields

On the deformed chiral superspace the covariant derivatives also have the standard expressions

$$
\begin{aligned}
D_\alpha &= \partial_\alpha + 2i(\sigma^m\bar\theta)_\alpha\partial_m\,, && (2.22)\\
\bar{D}_{\dot\alpha} &= -\bar\partial_{\dot\alpha}\,, && (2.23)
\end{aligned}
$$

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and satisfy

$$
\begin{aligned}
\{D_\alpha, \bar{D}_{\dot\alpha}\} &= -2i\sigma^m_{\alpha\dot\alpha}\partial_m\,, &(2.24)\\
\{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0\,, &(2.25)\\
\{D_\alpha, Q_\beta\} &= \{\bar{D}_{\dot\alpha}, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot\beta}\} = \{\bar{D}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0\,. &(2.26)
\end{aligned}
$$

Note that the change to the chiral superspace (2.5) was needed since otherwise D_α would not be a derivation with respect to the star product. Furthermore, because of the relations (2.26) we can use the covariant derivatives as in the classical case to define chiral and antichiral superfields. Chiral superfields are defined to satisfy $\hat{\bar{D}}_{\dot\alpha}\hat\Phi = 0$ and antichiral satisfy $\hat{D}_{\dot\alpha}\hat{\bar\Phi} = 0$, respectively. In components it is

$$
\begin{aligned}
\hat\Phi(\hat y, \hat\theta) &= \hat A(\hat y) + \sqrt{2}\hat\theta\hat\psi(\hat y) + \hat\theta\hat\theta\hat F(\hat y)\,, &(2.27)\\
\hat{\bar\Phi}(\hat{\bar y}, \hat{\bar\theta}) &= \hat{\bar A}(\hat{\bar y}) + \sqrt{2}\hat{\bar\theta}\hat{\bar\psi}(\hat{\bar y}) + \hat{\bar\theta}\hat{\bar\theta}\hat{\bar F}(\hat{\bar y}) &(2.28)
\end{aligned}
$$

where $\hat{\bar y}^m = \hat y^m - 2i\hat\theta\sigma^m\hat{\bar\theta}$. The vector superfield will be considered in the next section.

Two chiral superfields are multiplied using the star product (2.18). Clearly, the result is a function of y and θ and therefore it is a chiral superfield. Two antichiral superfields of the form (2.28) can be multiplied as

$$
\begin{aligned}
\bar\Phi_1(\bar y_1, \bar\theta) * \bar\Phi_2(\bar y_2, \bar\theta) &= \exp\left(2\bar\theta\bar\theta C^{ij}\frac{\partial}{\partial \bar y_1^i}\frac{\partial}{\partial \bar y_2^j}\right)\bar\Phi_1(\bar y_1, \bar\theta)\,\bar\Phi_2(\bar y_2, \bar\theta)\bigg|_{y_1, y_2 \to y}\\
&= \bar\Phi_1(\bar y, \bar\theta)\bar\Phi_2(\bar y, \bar\theta) + 2\bar\theta\bar\theta C^{ij}\frac{\partial}{\partial \bar y^i}\bar\Phi_1(\bar y, \bar\theta)\frac{\partial}{\partial \bar y^j}\bar\Phi_2(\bar y, \bar\theta)\,. \qquad (2.29)
\end{aligned}
$$

The result is an antichiral superfield.

2.4 Gauge theory and restriction of the gauge group

Consider the noncommutative gauge transformation of a chiral superfield $\hat\Phi$

$$\delta_\Lambda \Phi = -i\Lambda * \Phi\,, \qquad (2.30)$$

with the Lie algebra valued noncommutative gauge parameter $\hat\Lambda = \hat\Lambda_a T^a$ and $\hat{\bar{D}}\hat\Lambda = 0$ in order to preserve chirality. T^a are generators of the appropriate gauge group and form the Lie algebra

$$[T^a, T^b] = if^{ab}_{\ \ c}T^c\,. \qquad (2.31)$$

The commutator of two gauge transformations has the same form as in the classical case

$$\delta_\Lambda\delta_\Sigma - \delta_\Sigma\delta_\Lambda = \delta_{i[\Lambda \stackrel{*}{,} \Sigma]}\,, \qquad (2.32)$$

but the commutator

$$[\Lambda \stackrel{*}{,} \Sigma] = \frac{1}{2}\{\Lambda_a \stackrel{*}{,} \Sigma_b\}[T^a, T^b] + \frac{1}{2}[\Lambda_a \stackrel{*}{,} \Sigma_b]\{T^a, T^b\} \qquad (2.33)$$

only closes into the Lie algebra if the gauge group under consideration is $\mathrm{U}(N)$. This was already shown in the same way in [45] and in the context of instanton calculus in [42]. Thus in this setting gauge theories with gauge groups $\mathrm{SU}(N)$ can not be considered. However, using Seiberg-Witten map we can consider $\mathrm{SU}(N)$ or arbitrary groups.

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The $N = (1/2, 0)$ supersymmetric U(N) gauge theory is defined as follows [23, 46]. The gauge symmetry acts on the real Lie algebra valued vector superfield infinitesimally as

$$\delta_\Lambda V = -i\bar{\Lambda} * e_*^V + ie_*^V * \Lambda\,, \tag{2.34}$$

where the noncommutative exponential function is defined as

$$e_*^F = \sum_{n=0}^{\infty} \frac{1}{n!}\,(F)_*^n = 1 + F + \frac{1}{2}\,F * F + \frac{1}{6}\,F * F * F + \cdots \tag{2.35}$$

and Λ and $\bar{\Lambda}$ are matrices of chiral and antichiral superfields respectively. The noncommutative chiral and antichiral field strenghts

$$W_\alpha = -\frac{1}{4}\bar{D} * \bar{D}\, e_*^{-V} * D_\alpha * e_*^V\,, \tag{2.36}$$

$$\bar{W}_{\dot{\alpha}} = \frac{1}{4} D * D\, e_*^V * \bar{D}_{\dot{\alpha}} * e_*^{-V}\,, \tag{2.37}$$

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transform under (2.34) as

$$\delta_\Lambda W_\alpha = i[W_\alpha \overset{*}{,} \Lambda]\,, \tag{2.38}$$

$$\delta_\Lambda \bar{W}_{\dot{\alpha}} = i[\bar{W}_{\dot{\alpha}} \overset{*}{,} \bar{\Lambda}]\,, \tag{2.39}$$

Fixing the gauge freedom by a non(anti)commutative counterpart of the Wess-Zumino gauge, the vector superfield is reduced to

$$\begin{aligned} V(y, \theta, \bar{\theta}) = & -\theta\sigma^m\bar{\theta} v_m(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha\Big(\lambda_\alpha(y) + \frac{1}{2}\varepsilon_{\alpha\beta}C^{\beta\gamma}\sigma^m_{\gamma\dot{\gamma}}\bar{\lambda}^{\dot{\gamma}}v_m\Big) + \\ & + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\,(d(y) - i\partial_m v^m(y))\,, \end{aligned} \tag{2.40}$$

where, following [23], $\bar{\theta}\bar{\theta}\theta$ term is modified so that the standard gauge transformation rule follows for component fields. Then the remaining infinitesimal gauge symmetry (2.34), which preserves the gauge choice (2.40) is

$$\Lambda(y, \theta) = -\alpha(y)\,, \tag{2.41}$$

$$\bar{\Lambda}(\bar{y}, \bar{\theta}) = -\alpha(\bar{y}) - i\bar{\theta}\bar{\theta}C^{ij}(\partial_i\alpha v_j)(\bar{y})\,. \tag{2.42}$$

We also couple matter systhem by introducing a set of (anti)chiral superfields transforming in appropriate representations of the gauge group, i.e. the (anti)fundamental one

$$\delta_\Lambda\Phi = -i\Lambda * \Phi\,, \qquad \delta_\Lambda\bar{\Phi} = i\bar{\Phi} * \bar{\Lambda}\,. \tag{2.43}$$

Again, to ensure the standard gauge transformations of the component fields, we have to modify the antichiral field (2.26) as [46]:

$$\bar{\Phi}(\bar{y}, \bar{\theta}) = \bar{A}(\bar{y}) + \sqrt{2}\theta\bar{\psi}(\bar{y}) + \theta\theta\left(\bar{F}(\bar{y}) + iC^{ij}\partial_i\bar{A}v_j(\bar{y}) + \frac{1}{4}C^{ij}\bar{A}v_iv_j(\bar{y})\right). \tag{2.44}$$

Then the $N = (1/2, 0)$ supersymmetric gauge theory is described by the following Lagrange density

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{\text{int}} \tag{2.45}$$

where

$$\mathcal{L}_{YM} = \operatorname{Tr} W^\alpha * W_\alpha|_{\theta\theta} + \operatorname{Tr} \bar{W}_{\dot\alpha} * \bar{W}^{\dot\alpha}|_{\bar\theta\bar\theta}\,, \tag{2.46}$$

$$\mathcal{L}_{\text{int}} = \bar\Phi * e_*^{eV} * \Phi|_{\theta\theta\bar\theta\bar\theta}\,. \tag{2.47}$$

The component expansions of the Lagrange densities (2.46) and (2.46) are [23, 46]

$$\mathcal{L}_{YM} = \mathcal{L}_{YM}(C=0) - 2iC^{ij} \operatorname{Tr} f_{ij}\bar\lambda\bar\lambda + \frac{|C|^2}{2} \operatorname{Tr}(\bar\lambda\bar\lambda)^2\,, \tag{2.48}$$

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}(C=0) + \frac{i}{2}C^{ij} f_{ij}\bar{A}F - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^m_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}\psi_\beta\mathcal{D}_m\bar{A} - \frac{|C|^2}{16}\bar{A}\bar\lambda\bar\lambda F\,, \tag{2.49}$$

where

$$f_{ij} = \partial_i v_j - \partial_j v_i + \frac{i}{2}[v_i, v_j]\,, \tag{2.50}$$

$$\mathcal{D}_m\bar{A} = \partial_m\bar{A} + \frac{i}{2}v_m\bar{A}\,. \tag{2.51}$$

2.5 Construction of the Seiberg-Witten map in terms of component fields

To determine the Seiberg-Witten map we start with the Seiberg-Witten equation for the gauge superfield $\hat{V}$

$$\hat{V}(V) + \hat\delta_{\hat\Lambda}\hat{V}(V) = \hat{V}(V + \delta_\Lambda V)\,. \tag{2.52}$$

The hat indicate the dependence on the classical fields. To simplify matters we consider the abelian case and furthermore choose the Wess-Zumino gauge in which the superfields $\hat{V}$, $\hat\Lambda$ and $\bar{\hat\Lambda}$ have the forms (2.40), (2.41) and (2.42).

Following the procedure described in [6, 57] we expand first the superfields in the non(anti)commutative parameter $C^{\alpha\beta}$

$$\hat{V} = V + V'(V, C^{\alpha\beta}) + o(C^2)\,, \tag{2.53}$$

$$\hat\Lambda = \Lambda + \Lambda'(\Lambda, V, C^{\alpha\beta}) + o(C^2)\,, \tag{2.54}$$

and solve the Seiberg-Witten equation (2.52) perturbatively order by order in the noncommutativity parameter $C^{\alpha\beta}$. To zeroth order we get the classical gauge transformations. To first order we get

$$V' - V'(V + \delta_\Lambda V) + i(\Lambda' - \bar\Lambda') = \frac{i}{2}\, C^{\alpha\beta}\,\partial_\alpha V \partial_\beta(\Lambda + \bar\Lambda)\,, \tag{2.55}$$

From this equation we can read off the Seiberg-Witten equations for the component fields. With the assumption that the noncommutative component fields depend only on their classical counterparts and the classical gauge field v_m, we get the following Seiberg-Witten equations for component fields to first order in $C^{\alpha\beta}$

$$v'_m(v) - v'_m(v + \delta_\alpha v) - 2\,\partial_m \alpha'(\alpha, v) \;=\; 0\,, \tag{2.56}$$

$$\lambda'(v, \lambda) - \lambda'(v + \delta_\alpha v, \lambda) \;=\; 0\,, \tag{2.57}$$

$$\bar\lambda'(v, \bar\lambda) - \bar\lambda'(v + \delta_\alpha v, \bar\lambda) \;=\; 0\,, \tag{2.58}$$

$$d'(v, d) - d'(v + \delta_\alpha v, d) \;=\; 0\,, \tag{2.59}$$

since in the abelian case $\delta_\alpha\lambda = \delta_\alpha\bar\lambda = \delta_\alpha d = 0$.

These equations have the same form as the homogenous Seiberg-Witten equations in the case of canonically deformed superspace [61, 62, 57]. However, we can not simply take the known homogenous solutions from canonically deformed superspace since the mass dimension of the noncommutativity parameters differs.

With the help of dimensional analysis we can see that there are no local solutions of the equations (2.56)–(2.59). We will demonstrate this briefly with the Seiberg-Witten map for the gauge field v_m. The mass dimesions of v_m, the bosonic derivative ∂_m and the noncommutativity parameter C^{ij} are

$$[v_m] = [\partial_m] = 1\,, \qquad [C^{ij}] = -1\,. \tag{2.60}$$

For the Seiberg-Witten map for v'_m we make the ansatz

$$v'_m(C, v) = C^{ij} F_{ijm}(v, \partial v, \ldots)\,. \tag{2.61}$$

whereas $F_{ijm}(v, \partial v, \ldots)$ represent all possible local terms one can build from the gauge field v_m and its derivatives with the given index structure. Thus the mass dimension of $F_{ijm}(v, \partial v, \ldots)$ is

$$[F_{ijm}(v, \partial v, \ldots)] \geq 3\,, \tag{2.62}$$

and we get the wrong mass dimension for the field v'_m. From equation (2.53) it is obvious that v'_m has the same mass dimension as v_m. The same can be shown for α', λ', $\bar\lambda'$ and d'.

By considering equation (2.55), it is more naturale to assume that the noncommutative component fields v'_m, λ' etc. could depend on every classical field of the gauge supermultiplet and not only on their classical counterpart e.g.

$$v'_m = v'_m(v, \lambda, \bar\lambda, d, \partial v, \partial\lambda, \ldots)\,. \tag{2.63}$$

In this case there exist local Seibeerg-Witten maps with right mass dimension only for the field d' e.g.

$$d' = aC^{ij}\lambda\sigma_{ij}\lambda\,, \tag{2.64}$$

where a is a arbitrary constant.

One interesting question is if the nonlocal Seiberg-Witten maps lead to an action which is invariant under $N = 1/2$ supersymmetry. This question is automatically answered by solving the Seiberg-Witten equation in terms of superfields. For this we will apply the method developed by Wess and collaborators in [7]–[10] to determine the Seiberg-Witten maps for the superfield case.

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2.6 Construction of the Seiberg-Witten map in terms of superfields

We consider again the noncommutative gauge transformations of the chiral and antichiral matter fields (2.43), but with enveloping algebra valued gauge parameters $\hat{\Lambda}$ and $\hat{\bar{\Lambda}}$, e.g.

$$\hat{\Lambda} = \Lambda_a T^a + \Lambda^{'}_{ab} : T^a T^b : + \Lambda^{''}_{abc} : T^a T^b T^c : + \cdots , \tag{2.65}$$

where $\Lambda^{'}$ is linear in C^{ij}, $\Lambda^{''}$ is quadratic in C^{ij}, etc.The dots indicate that we have to sum over a basis of the vector space spanned by the homogenous polynomials in the generators T^a of the Lie algebra.

The commutator of two transformations (2.43) is certainly enveloping algebra valued. Hence we can use arbitrary Lie groups but the price we seem to have to pay is an infinite number of gauge parameters and an infinite number of gauge fields.

To avoid this problem we define new gauge transformations, where all these infinitely many gauge parameters depend just on the classical gauge parameter Λ or $\bar{\Lambda}$ respectively, the classical gauge field V and on their derivatives. We assume moreover that all superfields considered (e.g. $\hat{\Phi}$, $\hat{V}$) depend on their classical counterparts, the classical gauge field V and on their derivatives. This dependence, which we call Seiberg-Witten map, will be denoted by $\hat{\Lambda}(\Lambda, V)$, $\hat{\bar{\Lambda}}(\bar{\Lambda}, V)$, $\hat{\bar{\Phi}}(\Phi, V)$ and $\hat{V}(V)$.

The gauge transformations for the noncommutative chiral and antichiral matter fields and the noncommutative gauge field now have the form

$$\delta_\Lambda \hat{\Phi}(\Phi, V) \;=\; -i\hat{\Lambda}(\Lambda, V) * \hat{\Phi}(\Phi, V)\,, \tag{2.66}$$

$$\delta_\Lambda \hat{\bar{\Phi}}(\bar{\Phi}, V) \;=\; i\hat{\bar{\Phi}}(\Phi, V) * \hat{\bar{\Lambda}}(\bar{\Lambda}, V)\,, \tag{2.67}$$

$$\delta_\Lambda \hat{V}(V) \;=\; -i\hat{\bar{\Lambda}}(\bar{\Lambda}, V) * e_*^{\hat{V}(V)} + ie_*^{\hat{V}(V)} * \hat{\Lambda}(\Lambda, V)\,. \tag{2.68}$$

Since equation (2.32), called consistency condition in [10], involves solely the gauge parameters, it is convenient to base the construction of the Seiberg-Witten map on it and on the corresponding condition for the gauge parameters of the antifundamental representation. In a second step the remaining Seiberg-Witten maps for the matter fields and the gauge field can be computed from the equations (2.66)–(2.68).

The procedure in the abelian case is the following. As was mentioned, we start with the consistency conditions which have the following form in the abelian case

$$(\delta_\Lambda \delta_\Sigma - \delta_\Sigma \delta_\Lambda)\, \hat{\Phi}(\Phi, V) \;=\; 0\,, \tag{2.69}$$

$$(\delta_\Lambda \delta_\Sigma - \delta_\Sigma \delta_\Lambda)\, \hat{\bar{\Phi}}(\bar{\Phi}, V) \;=\; 0\,. \tag{2.70}$$

With equations (2.66) and (2.67) we get more explicitly

$$i\delta_\Lambda \hat{\Sigma}(\Sigma, V) - i\delta_\Sigma \hat{\Lambda}(\Lambda, V) + \hat{\Sigma}(\Sigma, V) * \hat{\Lambda}(\Lambda, V) - \hat{\Lambda}(\Lambda, V) * \hat{\Sigma}(\Sigma, V) \;=\; 0\,, \tag{2.71}$$

$$i\delta_\Lambda \hat{\bar{\Sigma}}(\Sigma, V) - i\delta_\Sigma \hat{\bar{\Lambda}}(\Lambda, V) + \hat{\bar{\Sigma}}(\Sigma, V) * \hat{\bar{\Lambda}}(\Lambda, V) - \hat{\bar{\Lambda}}(\Lambda, V) * \hat{\bar{\Sigma}}(\Sigma, V) \;=\; 0\,. \tag{2.72}$$

The variation $\delta_\Lambda \hat{\Sigma}(\Sigma, V)$ refers to the V-dependence of $\hat{\Sigma}(\Sigma, V)$ and the gauge transformation of the supersymmetric abelian gauge field V

$$\delta_\Lambda V = i(\Lambda - \bar{\Lambda})\,. \tag{2.73}$$

We now expand the consistency conditions and the gauge transformations (2.66)–(2.68) using the expansions (2.53), (2.54), the corresponding expansions for $\hat{\bar{\Lambda}}$, $\hat{\Phi}$, $\hat{\bar{\Phi}}$ and the expanded star product (2.18). From these expanded equations we can then determine the Seiberg-Witten maps order by order in C^{ij} for all considered superfields. To first order in C^{ij} we get the following equations

$$\delta_\Lambda \Sigma^{'}(\Sigma, V) - \delta_\Sigma \Lambda^{'}(\Lambda, V) = iC^{\alpha\beta}\partial_\alpha \Lambda \partial_\beta \Sigma\,, \tag{2.74}$$

$$\delta_\Lambda \bar{\Sigma}^{'}(\bar{\Sigma}, V) - \delta_\Sigma \bar{\Lambda}^{'}(\bar{\Lambda}, V) = iC^{\alpha\beta}\partial_\alpha \bar{\Lambda} \partial_\beta \bar{\Sigma}\,, \tag{2.75}$$

$$\delta_\Lambda \Phi^{'}(\Phi, V) + i\Lambda\Phi^{'}(\Phi, V) + i\Lambda^{'}(\Lambda, V)\Phi = \frac{i}{2}C^{\alpha\beta}\partial_\alpha \Lambda \partial_\beta \Phi \tag{2.76}$$

$$\delta_\Lambda \bar{\Phi}^{'}(\bar{\Phi}, V) - i\bar{\Lambda}\bar{\Phi}^{'}(\bar{\Phi}, V) + i\bar{\Lambda}^{'}(\bar{\Lambda}, V)\bar{\Phi} = \frac{i}{2}C^{\alpha\beta}\partial_\alpha \bar{\Lambda} \partial_\beta \bar{\Phi} \tag{2.77}$$

$$\delta_\Lambda V^{'}(V) - i\Lambda^{'}(\Lambda, V) + i\bar{\Lambda}^{'}(\bar{\Lambda}, V) = \frac{i}{2}C^{\alpha\beta}\partial_\alpha (\Lambda + \bar{\Lambda}) \partial_\beta V\,. \tag{2.78}$$

We will now look for solutions of these equations.

2.7 Seiberg-Witten map for the gauge parameters

For the gauge parameter Λ' there exist a local, chiral and $N = (1/2, 0)$ supersymmetric Seiberg-Witten map. It is

$$\Lambda^{'}(\Lambda, V) = -\frac{1}{2}C^{\alpha\beta}\partial_\alpha \Lambda \partial_\beta M V\,, \tag{2.79}$$

where M is the $N = (1/2, 0)$ chiral projector [57]

$$M = \frac{1}{16\Box}\bar{D}^2 D^\alpha M_\alpha\,, \tag{2.80}$$

with

$$M_\alpha(y) = -\partial_\alpha + 2i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m\,, \tag{2.81}$$

For the gauge parameter $\bar{\Lambda}'$ however, there does not exist a local, antichiral and $N = (1/2, 0)$ supersymmetric solution. This can be shown using dimensional analysis.

The right hand side of the consistency condition (2.75) is linear in each of the classical superfields $\bar{\Lambda}$ and $\bar{\Sigma}$. All terms in the ansatz for $\bar{\Lambda}^{'}$ which would contain powers of V can therefore solve only the homogeneous consistency condition because of (2.73). Hence we make an ansatz for $\bar{\Lambda}^{'}$ only linear in V without loss of generality. Moreover $\bar{\Lambda}^{'}$ has to be linear in the classical gauge parameter $\bar{\Lambda}$ and by definition linear in C^{ij} or $C^{\alpha\beta}$ respectively. In order to preserve the $N = (1/2, 0)$ supersymmetry we may only use the bosonic derivatives ∂_m, the spinor derivative ∂_α, the spinor coordinate $\bar{\theta}$ and the covariant spinor derivatives (2.22) and (2.23). The mass dimensions of these objects are

$$[\bar{\Lambda}^{'}] = [\bar{\Lambda}] = [V] = 0\,, \qquad [C^{\alpha\beta}] = -1\,, \qquad \partial_m = 1$$
$$[\partial_\alpha] = \frac{1}{2}\,, \qquad [\bar{\theta}] = -\frac{1}{2}\,, \qquad [D_\alpha] = [\bar{D}_{\dot{\alpha}}] = \frac{1}{2}\,. \tag{2.82}$$

It is not hard to see that there is only one term built out of this objects with mass dimension zero and which is local, antichiral, preserve the $N = (1/2, 0)$ supersymmetry and have appropriate index contraction. It is

$$\bar{\Lambda}' = aC^{\alpha\beta}\partial_\alpha\bar{\Lambda}D^2\bar{\theta}\bar{\theta}V\,. \tag{2.83}$$

where a is a constant. It is obvious that there is no choice of this constant such that (2.83) is a solution of the equation (2.75). Thus we have shown that for $\bar{\Lambda}'$ there is no local and antichiral solution which preserve the $N = (1/2, 0)$ supersymmetry.

Using the same method one could show that even if we give up antichirality there is no local solution of equation (2.75) which preserve the $N = (1/2, 0)$ supersymmetry. If we give up the $N = (1/2, 0)$ supersymmetry there exists a local and antichiral expression, namely

$$\bar{\Lambda}'(\Lambda, V) = \frac{1}{2}C^{\alpha\beta}\partial_\alpha\bar{\Lambda}\partial_\beta\bar{M}V\,, \tag{2.84}$$

which solves the equation (2.75). $\bar{M}$ is the $N = (0, 1/2)$ antichiral projector [57]

$$\bar{M} = \frac{1}{16\Box}D^2\bar{D}_{\dot\alpha}\bar{M}^{\dot\alpha}\,, \tag{2.85}$$

with

$$\bar{M}^{\dot\alpha}(y) = -\bar{\partial}^{\dot\alpha} + 4i\bar{\sigma}^{m\,\dot\alpha\alpha}\theta_\alpha\partial_m\,. \tag{2.86}$$

Nonlocal Seiberg-Witten maps for Λ' and $\bar{\Lambda}'$ are

$$\Lambda'(\Lambda, V) = -\frac{1}{2}C^{\alpha\beta}\partial_\alpha\Lambda\partial_\beta PV\,, \tag{2.87}$$

$$\bar{\Lambda}'(\bar{\Lambda}, V) = \frac{1}{2}C^{\alpha\beta}\partial_\alpha\bar{\Lambda}\partial_\beta\bar{P}V\,. \tag{2.88}$$

where P and $\bar{P}$ are the covariant chiral and antichiral superfield projectors

$$P = \frac{\bar{D}^2D^2}{16\Box}\,, \qquad \bar{P} = \frac{D^2\bar{D}^2}{16\Box}\,. \tag{2.89}$$

These solutions preserve the $N = (1/2, 0)$ supersymmetry and are chiral and antichiral, respectively. The component expansion of these solutions in the Wess-Zumino gauge are

$$\Lambda' = 0\,, \tag{2.90}$$

$$\bar{\Lambda}' = \bar{\theta}\bar{\theta}C^{ij}\partial_i\alpha\frac{\partial_j}{\Box}(d - i\partial^m v_m)\,. \tag{2.91}$$

We will now look for solutions of the equations (2.76)–(2.78).

2.8 Seiberg-Witten maps for the matter and gauge fields

It is not hard to find solutions of the equations (2.76)–(2.78) which correspond to the nonlocal solutions for Λ' (2.85) and $\bar{\Lambda}'$ (2.86). They are

$$\Phi'(\Phi, V) = -\frac{1}{2}C^{\alpha\beta}\partial_\alpha\Phi\partial_\beta PV\,, \tag{2.92}$$

$$\bar{\Phi}'(\bar{\Phi}, V) = \frac{1}{2}C^{\alpha\beta}\partial_\alpha\bar{\Phi}\partial_\beta\bar{P}V\,, \tag{2.93}$$

$$V'(V) = -\frac{1}{2}C^{\alpha\beta}\partial_\alpha V\partial_\beta PV + \frac{1}{2}C^{\alpha\beta}\partial_\alpha V\partial_\beta\bar{P}V - \frac{1}{2}C^{\alpha\beta}\partial_\alpha PV\partial_\beta\bar{P}V\,. \tag{2.94}$$

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The component expansion of these superfields in Wess-Zumino gauge are

$$\Phi'(y,\theta) = A'(y) + \sqrt{2}\theta\psi'(y)\,, \tag{2.95}$$

$$\bar{\Phi}'(y,\bar\theta) = \bar\theta\bar\theta\bar{F}'(y)\,, \tag{2.96}$$

$$V'(y,\theta,\bar\theta) = \bar\theta\bar\chi'(y) + \bar\theta\bar\theta M(y) - \theta\sigma^m\bar\theta v'_m(y) - i\bar\theta\bar\theta\theta\lambda'(y)\,, \tag{2.97}$$

where the Seiberg-Witten maps for the component fields are

$$A'(\psi,\bar\lambda) = \frac{1}{\sqrt{2}}C^{ij}\,\psi\sigma_j\frac{\partial_i}{\Box}\bar\lambda\,, \tag{2.98}$$

$$\psi'_\alpha(F,\bar\lambda) = C^{ij}\left(\sigma_j\frac{\partial_i}{\Box}\bar\lambda\right)_\alpha F\,, \tag{2.99}$$

$$\bar{F}'(A,v,d) = -C^{ij}\partial_i\alpha\frac{\partial_j}{\Box}(d - i\partial^m v_m)\,, \tag{2.100}$$

$$\bar\chi'^{\dot\alpha}(v,\bar\lambda,d) = C^{ij}\left(\bar\sigma^m\sigma_j\frac{\partial_i}{\Box}\bar\lambda\right)^{\dot\alpha}\left(\frac{\partial_m}{\Box}(d - i\partial^n v_n) - \frac{1}{2}v_m\right), \tag{2.101}$$

$$M'(v,\lambda,\bar\lambda,d) = -\frac{i}{2}C^{ij}\left(\lambda\sigma_j\frac{\partial_i}{\Box}\bar\lambda + \frac{1}{2}v_j\frac{\partial_i}{\Box}(d - i\partial^n v_n)\right), \tag{2.102}$$

$$v'_m(\bar\lambda) = \frac{i}{2}C^{ij}\frac{\partial_i}{\Box}\bar\lambda\bar\sigma_j\sigma_m\bar\lambda\,, \tag{2.103}$$

$$\lambda'_\alpha(v,\bar\lambda,d) = \frac{i}{2}C^{ij}\left(\sigma_j\frac{\partial_i}{\Box}\bar\lambda(d - i\partial^n v_n) + \frac{1}{2}\sigma_j\bar\lambda\frac{\partial_i}{\Box}(d - i\partial^n v_n)\right). \tag{2.104}$$

As expected, this maps are nonlocal and did not coincide with the original ones proposed by Seiberg and Witten when the superpartner fields are set to zero.

The Seiberg-Witten maps for the matter and gauge fields which correspond to the solutions (2.79) and (2.84) are obtained by replacing P with M and $\bar{P}$ with $\bar{M}$ in equations (2.92)–(2.94). In Wess-Zumino gauge these maps simplify to

$$\Phi'(\Phi,V) = 0\,, \tag{2.105}$$

$$\bar\Phi'(\bar\Phi,V) = \frac{1}{2}C^{\alpha\beta}\partial_\alpha\bar\Phi\partial_\beta\bar{M}V\,, \tag{2.106}$$

$$V'(V) = \frac{1}{2}C^{\alpha\beta}\partial_\alpha V\partial_\beta\bar{M}V\,, \tag{2.107}$$

since in this gauge $\partial_\alpha MV = 0$. It is clear that these solutions breke tha $N = (1/2, 0)$ supersymmetry because of $\{Q, \bar{M}\} \neq 0$. Furthermore this solutions are nonlocal as well. For this reasons we will not consider them further.

We come now to the Seiberg-Witten maps for the field strengths and the U(1) Yang-Mills Lagrange density.

2.9 Seiberg-Witten map for the field strengths and Yang-Mills action

The expansion of the gauge field (2.53) induce a expansion of the fieldstrengths in terms of the non(anti)commutative parameter $C^{\alpha\beta}$

$$\hat{W}_\alpha = W_\alpha + W'_\alpha(V, C^{\alpha\beta}) + o(C^2)\,, \tag{2.108}$$

$$\hat{\bar{W}}_{\dot\alpha} = \bar{W}_{\dot\alpha} + \bar{W}'_{\dot\alpha}(V, C^{\alpha\beta}) + o(C^2)\,. \tag{2.109}$$

The Seiberg-Witten map for the field strengths can be determined from equations (2.36) and (2.37). In the Wess-Zumino gauge we get to first order in $C^{\alpha\beta}$

$$W'_\alpha(V) = -\frac{1}{4}\bar{D}^2\left(D_\alpha V' + \frac{1}{2}C^{\beta\gamma}\partial_\beta D_\alpha V \partial_\beta V\right), \tag{2.110}$$

$$\bar{W}'_{\dot\alpha}(V) = -\frac{1}{4}D^2\left(D_{\dot\alpha} V' - \frac{1}{2}C^{\beta,\gamma}\partial_\beta D_{\dot\alpha} V \partial_\beta V\right). \tag{2.111}$$

It is obvious that these Seiberg-Witten maps are also nonlocal. With (2.97) we obtain for the expansion of W'_α and $\bar{W}'_{\dot\alpha}$ in terms of clasical component fields

$$\begin{aligned} W'_\alpha(y,\theta) &= -i\lambda'_\alpha - i(\sigma^m\partial_m\bar\chi')_\alpha - i(\sigma^m\bar\sigma^n\theta)_\alpha\partial_m v'_n + \\ &\quad + \frac{i}{2}C^{ij}(\sigma_j\bar\lambda)_\alpha v_i + \frac{1}{2}C^{ij}(\sigma_{ij}\theta)_\alpha\bar\lambda\bar\lambda\,, \end{aligned} \tag{2.112}$$

$$\begin{aligned} \bar{W}'_{\dot\alpha}(\bar y,\bar\theta) &= -i(\bar\theta\bar\sigma^m\sigma^n)_{\dot\alpha}\partial_m v'_n + \\ &\quad + \bar\theta\bar\theta\left(\Box\bar\chi'_{\dot\alpha} + (\partial_m\lambda'\sigma^m)_{\dot\alpha} + C^{ij}\partial_j(\bar\lambda_{\dot\alpha}v_i) - \frac{1}{2}C^{ij}\partial_m(v_i\bar\lambda\bar\sigma^m\sigma_j)_{\dot\alpha}\right), \end{aligned} \tag{2.113}$$

where $\bar\chi'$, v'_m and λ' are given in (2.101), (2.103) and (2.104).

The Yang-Mills Lagrange density expanded up to first order in C^{ij} is

$$\begin{aligned} \mathcal{L}_{YM} &= \hat{W}^\alpha * \hat{W}_\alpha|_{\theta\theta} + \hat{\bar{W}}_{\dot\alpha} * \hat{\bar{W}}^{\dot\alpha}|_{\bar\theta\bar\theta} \\ &= \mathcal{L}_{YM}(C=0) + 2W^\alpha W'_\alpha|_{\theta\theta} + 2\bar{W}'_{\dot\alpha}\bar{W}^{\dot\alpha}|_{\bar\theta\bar\theta} + o(C^2)\,. \end{aligned} \tag{2.114}$$

With the equations (2.112) and (2.113) we obtain up to total derivatives

$$\begin{aligned} W^\alpha W'_\alpha|_{\theta\theta} &= i\bar\lambda\Box\bar\chi' + i\partial_m\lambda'\sigma^m\bar\lambda + id\partial^m v'_m - \frac{1}{2}f^{SD}_{ab}f'^{ab} - \\ &\quad - \frac{i}{2}C^{ij}\partial_m\bar\lambda\bar\sigma^m\sigma_j\bar\lambda v_i - \frac{i}{2}C^{ij}f_{ij}\bar\lambda\bar\lambda\,, \end{aligned} \tag{2.115}$$

$$\begin{aligned} \bar{W}'_{\dot\alpha}\bar{W}^{\dot\alpha}|_{\bar\theta\bar\theta} &= i\bar\lambda\Box\bar\chi' + \partial_m\lambda'\sigma^m\bar\lambda + id\partial^m v'_m - \frac{1}{2}f^{ASD}_{ab}f'^{ab} + \\ &\quad + \frac{i}{2}C^{ij}\bar\lambda\bar\sigma^m\sigma_j\partial_m\bar\lambda v_i - \frac{i}{2}C^{ij}f_{ij}\bar\lambda\bar\lambda\,, \end{aligned} \tag{2.116}$$

where $f'_{ab} = \partial_a v'_b - \partial_b v'_a$ and f^{SD}_{ab} and f^{ASD}_{ab} are the selfdual and antiselfdual field strengths, respectively. It is obvious that $\mathcal{L}_{YM}$ (2.114) is nonlocal due to the fields $\bar\chi'$, λ' and v'_m. It is also worth noticing, that this Lagrange density has also the same term as the Lagrange density (2.47).

The Lagrange density (2.114) differs from the Lagrange density (2.47) by one local and three nonlocal terms. Thus, the Lagrange density (2.114) is nonlocal, as expected.

3. $N = (1/2, 1/2)$ deformed euclidean superspace

We consider now the following deformed superspace [27, 54]:

$$\begin{aligned} & [\hat x^i, \hat x^j] = \bar\theta\bar\theta C^{ij}\,, \\ \hat{\mathcal{R}}:\ & [\hat x^i, \hat\theta^\alpha] = -iC^{\alpha\beta}\sigma^m_{\beta\dot\beta}\hat{\bar\theta}^{\dot\beta}\,, \qquad [\hat x^i, \hat{\bar\theta}^{\dot\beta}] = 0\,, \\ & \{\hat\theta^\alpha, \hat\theta^\beta\} = C^{\alpha\beta}\,, \qquad \{\hat{\bar\theta}^{\dot\alpha}, \hat{\bar\theta}^{\dot\beta}\} = \{\hat\theta^\alpha, \hat{\bar\theta}^{\dot\alpha}\} = 0\,, \end{aligned} \tag{3.1}$$

where the equations (2.2)–(2.4) still hold. The noncommutative functions, fields and derivatives are again defined as in (2.6)–(2.9). The algebra (3.1) is also covariant under the group of classical supertranslations (2.10).

It is convenient to use the chiral coordinates $y^m = x^m + i\theta\sigma^m\bar\theta$ instead of x^m. One may readily check that the (anti)commutation relations (3.1) become

$$
\begin{aligned}
&[\hat y^i, \hat y^j] = 4\bar\theta\bar\theta C^{ij}\,, \\
\hat{\mathcal{R}}:\ &[\hat y^i, \hat\theta^\alpha] = -2iC^{\alpha\beta}\sigma^m_{\beta\dot\beta}\hat{\bar\theta}^{\dot\beta}\,, \qquad\qquad [\hat y^i, \hat{\bar\theta}^{\dot\beta}] = 0\,, \\
&\{\hat\theta^\alpha, \hat\theta^\beta\} = C^{\alpha\beta}\,, \qquad\qquad \{\hat{\bar\theta}^{\dot\alpha}, \hat{\bar\theta}^{\dot\beta}\} = \{\hat\theta^\alpha, \hat{\bar\theta}^{\dot\alpha}\} = 0\,.
\end{aligned} \tag{3.2}
$$

On this deformed chiral superspace the generators of supertranslation and the covariant deivatives have the same form and the same anticomutation relations as in sections 2.1 and 2.3. The chiral and anichiral superfields are also defined in standard way, namely $\hat{\bar D}_{\dot\alpha}\hat\Phi = 0$ and $\hat D_{\dot\alpha}\hat{\bar\Phi} = 0$.

The star product is given by

$$
F(\theta) * G(\theta) = \exp\left(-\frac{1}{2}C^{\alpha\beta}D^F_\alpha D^G_\beta\right) F(\theta)\, G(\theta) \tag{3.3}
$$

where D^F_α acts only on F and D^G_β acts only on G, e.g.

$$
D^G_\beta(FG) = (-1)^{p(F)} D_\beta G\,. \tag{3.4}
$$

Although the star product (3.3) preserves the $N = (1/2, 1/2)$ supersymmetry, it does not allow to define chiral superfields which form subalgebras of the star product. For this reson the deformation (3.2) leads for the Wess-Zumino model to the same lagrangian as the deformation (2.5), preserving only the $N = (1/2, 0)$ supersymmetry [27, 54].

The gauge theory can be constructed following section 2.4. The equations (2.30)–(2.39) still hold. Moreover, in [54] it was demonstrated that also the Yang-Mills actions on both superspaces are equivalent. Because of equation (2.33) we can consider again only gauge theories with U(N) gauge groups. To consider gauge theories with SU(N) gauge groups we have to determine the Seiberg-Witten map.

3.1 Construction of the Seiberg-Witten maps in terms of superfields

The steps in constructing the Seiberg-Witten maps are the same as in section2.6. The equations (2.65)–(2.73) did not change. We consider here again the abelian case. With the star product (3.3) we get the following equations from which the Seiberg-Witten maps to first order in C^{ij} can be determined:

$$
\delta_\Lambda\Sigma'(\Sigma, V) - \delta_\Sigma\Lambda'(\Lambda, V) = iC^{\alpha\beta}D_\alpha\Lambda D_\beta\Sigma\,, \tag{3.5}
$$

$$
\delta_\Lambda\bar\Sigma'(\bar\Sigma, V) - \delta_\Sigma\bar\Lambda'(\bar\Lambda, V) = 0\,, \tag{3.6}
$$

$$
\delta_\Lambda\Phi'(\Phi, V) + i\Lambda\Phi'(\Phi, V) + i\Lambda'(\Lambda, V)\Phi = \frac{i}{2}C^{\alpha\beta}D_\alpha\Lambda D_\beta\Phi\,, \tag{3.7}
$$

$$
\delta_\Lambda\bar\Phi'(\bar\Phi, V) - i\bar\Lambda\bar\Phi'(\bar\Phi, V) + i\bar\Lambda'(\bar\Lambda, V)\bar\Phi = 0\,, \tag{3.8}
$$

$$
\delta_\Lambda V'(V) - i\Lambda'(\Lambda, V) + i\bar\Lambda'(\bar\Lambda, V) = \frac{i}{2}C^{\alpha\beta}D_\alpha\Lambda D_\beta V\,. \tag{3.9}
$$

We will now look for solutions of these equations.

3.2 Seiberg-Witten maps for gauge parameters

First of all we have to find solutions for $\Lambda^{'}$ and $\bar{\Lambda}'$ since the Seiberg-Witten maps for the matter fields and the gauge field depend on them. For $\bar{\Lambda}'$ we can simply take

$$\bar{\Lambda}'(\Lambda, V) = 0\,, \tag{3.10}$$

which is a solution of the equation (3.6). The other solutions of the equation (3.6) either breake the $N = (1/2, 1/2)$ supersymmetry or locality. This is obvious from dimensional analysis. The mass dimension of $\bar{\Lambda}^{'}$ is zero (2.82). The Seiberg-Witten map for $\bar{\Lambda}^{'}$ contains at least $\bar{\Lambda}$, $C^{\alpha\beta}$ or C^{ij} respectively and the gauge invariant field strengths W_α or $\bar{W}_{\dot{\alpha}}$. The mass dimension of these three objects is

$$[C\Lambda W] = \frac{1}{2}\,. \tag{3.11}$$

In order to get the right mass dimension we have to use operators with negativ mass dimension. One posibility is to use $\bar{\theta}\bar{\theta}$, e.g.

$$\bar{\Lambda}^{'}(\bar{\Lambda}, W) \sim C^{\alpha\beta}\bar{\theta}\bar{\theta}\bar{\Lambda} D_\alpha W_\beta\,, \tag{3.12}$$

but these solutions possess only $N = (1/2, 0)$ symmetry. The other posibility is to use nonlocal operators, e.g

$$\bar{\Lambda}^{'}(\bar{\Lambda}, \bar{W}) \sim C^{\alpha\beta}\sigma^m_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Lambda}\frac{\partial_m}{\Box}W_\beta\,. \tag{3.13}$$

It is not hard to see that the Seiberg-Witten equations for $\bar{\Lambda}''$, $\bar{\Lambda}'''$, ..., are of homogenous type like equation (3.6), e.g.

$$\delta_\Lambda\bar{\Sigma}^{''}(\bar{\Sigma}, V) - \delta_\Sigma\bar{\Lambda}^{''}(\bar{\Lambda}, V) = 0\,. \tag{3.14}$$

This is the case because of the star product (3.3). Thus we can take the trivial solution for all these fields and get

$$\bar{\hat{\Lambda}} = \bar{\Lambda}\,, \tag{3.15}$$

which means that the antichiral gauge parameter remains undeformed.

We come now to the solutions for $\Lambda^{'}$. From equation (3.5) it is obvious that $\Lambda^{'}$ must contain the nonchiral term $D_\alpha\Lambda$. Thus there does not exist a chiral Seiberg-Witten map for $\Lambda^{'}$ which solves equation (3.5). A nochiral, local and $N = (1/2, 1/2)$ supersymmetric solution is

$$\Lambda^{'}(\Lambda, V) = -\frac{1}{2}C^{\alpha\beta}D_\alpha\Lambda D_\beta V\,. \tag{3.16}$$

The component expansion of this solution is

$$\bar{\Lambda}^{'} = C^{ij}\bar{\theta}\bar{\theta}\left(iv_i\partial_j\alpha + \theta\sigma_j\bar{\lambda}\partial_i\alpha\right). \tag{3.17}$$

We can find easily further solutions for $\Lambda^{'}$ using the chiral projectors P (2.89) and M (2.80). These are

$$\Lambda^{'}(\Lambda, V) = -\frac{1}{2}C^{\alpha\beta}D_\alpha\Lambda D_\beta PV\,, \tag{3.18}$$

$$\Lambda^{'}(\Lambda, V) = -\frac{1}{2}C^{\alpha\beta}D_\alpha\Lambda D_\beta MV\,. \tag{3.19}$$

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These solutions are not only nonchiral but also nonlocal, since

$$PV(y) = \frac{1}{2\Box}(d - i\partial^m v_m) - \theta\sigma^m \frac{\partial_m}{\Box}\bar{\lambda}\,, \tag{3.20}$$

$$MV(y) = -\frac{1}{\Box}(d - i\partial^m v_m)\,. \tag{3.21}$$

Moreover, the solution (3.21) possesses just the $N = (1/2, 0)$ supersymmetry.

Now we want to find solutions for the equations (3.7)–(3.9) which correspond to the solution (3.10) and to the solutions (3.16), (3.18) and (3.19), respectively. It is clear that in this case equations (3.7) and (3.9) become

$$\delta_\Lambda \Phi'(\Phi, V) + i\Lambda\Phi'(\Phi, V) = \frac{i}{2} C^{\alpha\beta} D_\alpha \Lambda D_\beta \Phi\,, \tag{3.22}$$

$$\delta_\Lambda V'(V) - i\Lambda'(\Lambda, V) = \frac{i}{2} C^{\alpha\beta} D_\alpha \Lambda D_\beta V\,. \tag{3.23}$$

3.3 Seiberg-Witten maps for matter and gauge fields

The Seiberg-Witten map for $\bar{\Phi}'$ which corresponds to the solution (3.10) is simply

$$\bar{\Phi}'(\bar{\Phi}, V) = 0\,. \tag{3.24}$$

For the field $\bar{\Phi}'$ it can be shown in exactly the same manner as for the field $\bar{\Lambda}'$ in previous section, that we can take the trivial Seiberg-Witten map to all orders in $C^{\alpha\beta}$

$$\bar{\Phi}'' = \bar{\Phi}''' = \bar{\Phi}'''' = \cdots = 0\,, \tag{3.25}$$

and get

$$\hat{\bar{\Phi}} = \bar{\Phi}\,, \tag{3.26}$$

which means that the antichiral matter field remain undeformed.

Among the solutions for Φ' and V' which correspond to the solution (3.16) for Λ', there are also the trivial solutions

$$\Phi'(\Phi, V) = V'(V) = 0\,. \tag{3.27}$$

Thus, in this case to first order in $C^{\alpha\beta}$ only the gauge parameter field is deformed whereas all other fields remain undeformed.

The Seiberg-Witten maps for Φ' and V' which correspond to the solution (3.18) are

$$\Phi'(\Phi, V) = -\frac{1}{2} C^{\alpha\beta} D_\alpha \Phi D_\beta PV\,, \tag{3.28}$$

$$V'(V) = -\frac{1}{2} C^{\alpha\beta} D_\alpha V D_\beta PV\,, \tag{3.29}$$

and which correspond to the solution (3.19) are

$$\Phi'(\Phi, V) = -\frac{1}{2} C^{\alpha\beta} D_\alpha \Phi D_\beta MV\,, \tag{3.30}$$

$$V'(V) = -\frac{1}{2} C^{\alpha\beta} D_\alpha V D_\beta MV\,. \tag{3.31}$$

respectively. It is obvious that all this solutions are nonlocal due to equations (3.20) and (3.21). Furthermore, the Seiberg-Witten maps (3.28) and (3.30) for the matter field Φ' are nonchiral and the later possesses only the $N = (1/2, 0)$ supersymmetry.

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4. Conclusions

We have considered Seiberg-Witten maps for superfields on the $N = (1/2, 0)$ and the $N = (1/2, 1/2)$ deformed superspaces. We have shown that on the $N = (1/2, 0)$ deformed superspace there is no Seiberg-Witten map for antichiral superfields which is at the same time antichiral, local and which preserve the $N = (1/2, 0)$ supersymmetry. Solutions wich break this requirements were presented.

On the $N = (1/2, 1/2)$ deformed superspace we have shown that for the chiral gauge parameter, and therefore also for the chiral matter field, there is no chiral Seiberg-Witten map. Some other possible Seiberg-Witten maps for the superfields were presented. It is worth noticing that the antichiral fields remain undeformed on this superspace.

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